ARTICLE IN PRESS

Knowledge-Based Systems xxx (xxxx) xxx

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A second-order-based decision tool for evaluating decisions under conditions of severe uncertainty☆

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ARTICLE INFO

Article history:
Received 28 February 2019
Received in revised form 3 October 2019
Accepted 8 November 2019
Available online xxxx

Keywords:
Decision analysis
Decision software
Imprecise criteria weights
Imprecise probabilities

ABSTRACT

The requirement to assign precise numerical values to model entities such as criteria weights, probabilities, and utilities is too strong in most real-life decision situations, and hence alternative representations and evaluation mechanisms are important to consider. In this paper, we discuss the *Decide*IT 3.0 state-of-the-art software decision tool and demonstrate its functionality using a real-life case. The tool is based on a belief mass interpretation of the decision information, where the components are imprecise by means of intervals and qualitative estimates, and we discuss how multiplicative and additive aggregations influence the resulting distribution over the expected values.

1. Introduction

There have been many suggestions for how to deal with the strong requirements of most decision models to provide precise information. One basic idea is to simply assign homogenous distributions over the variables, but this is done to the cost of significant information loss, which is why it is preferred to at least ordinally or cardinally rank the components such as probabilities, utilities, and criteria weights, and thereafter utilise one of several techniques to handle them. A quite useful approach is to utilise surrogate weights, derived from the rankings, and several authors have suggested various means for that, such as [1–5], and many others. The rankings are then transformed into numerical weights by dedicated mapping functions. Despite some controversies, many surrogate weight methods have been suggested, such as in [6] rank-sum weights and rank reciprocal weights), ROC (rank order centroid), e.g., [7]. Entropy arguments also occur, such as [4,8,9]. Yager [10] used an ordered weighting average method and [11,12] used the Maximum Entropy OWA (MEOWA) method based on minimal variability. The MEOWA is, however, a more complex measure than, e.g., ROC, and requires an attitude (similar to a pessimism–optimism) parameter. In many situations, there is a need utilise entirely different frameworks for representing vagueness, such as the theory of capacities, sets of probability measures, interval probabilities, evidence and possibility theories, fuzzy measures, preference rankings, and higher-order probability theory, or combinations of rankings with other representation formats (see for example [13–19] to name just a few in the extensive literature in the fields). Often, for these theories to be reasonably transparent to the decision-maker, (s)he is required to possess significant mathematical knowledge, and even then sometimes the theories include relatively harsh (and hence non-transparent) methods for discriminating between decision alternatives. Furthermore, the computational complexity can be high in various respects, as we have argued (for an extensive background, see, e.g., [20]), and there is a strong need for user-friendly tool support, while still maintaining a high capacity for evaluations of a wide range of assessment types.

We have during the last 20 years created evaluation software for these purposes. Our earlier versions of decision support software have been successfully used in a wide variety of contexts, e.g., long-term storage of nuclear waste, land use planning, choice of insurance portfolios against catastrophe events, massive-scale energy policy formation, gold mining evaluations, health-care planning, assessments for medical risks, emergency management, and so on [21–27].

In this paper, we demonstrate a significantly extended software tool *Decide*IT 3.0 [28], implementing our latest findings regarding aggregations of distributions. This is a landmark in the handling of imprecise information and differs significantly from earlier versions of the tool which handled only a pure interval

☆ No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to https://doi.org/10.1016/j.knosys.2019.105219.

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https://doi.org/10.1016/j.knosys.2019.105219
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approach with orderings. In the earlier versions, the result of an evaluation still contained unnecessary uncertainty regarding the final outcome. This is due to overlaps in expected values making it difficult to discriminate between the alternative options involved. By adding second-order information in the way done in *Decide*IT 3.0, we can significantly enhance a decision-maker's understanding of decision situations when handling aggregations of imprecise representations.

We will thus in this paper discuss the underlying framework enabling evaluations that are subject to incomplete input data. The software is able to evaluate decision situations including imprecise utilities, probabilities, and criteria weights, and qualitative estimates between these components. We avoid the introduction of new concepts into the decision models, such as set membership functions or similar formalisms, and instead use higher-order distributions of belief in the basic utilities, probabilities, and criteria weights which then allows for better and more transparent discrimination between the resulting values of the decision alternatives. The ability to use ordinary belief distributions over probabilities, values, and criteria weights enhances the transparency of the results since no new concepts have to be introduced in the evaluation of the model. There are no other known software tools (including earlier versions of *Decide*IT) that are able to calculate the resulting belief in expected values based on user input in terms of belief in probabilities, values, and criteria weights.

The next sections cover the decision-theoretical framework and explain the data model, and are followed by a description of an application to a real-life problem involving the largest Swedish energy provider. The presentation starts with the representation model in which the decision data is structured and stored. The tool supports elicitation models as discussed in [29]. Having successfully elicited and stored the decision data in the tool, the evaluation can commence. Thus, the evaluation model is described next. To illuminate the entire process, a real-life example concludes the paper, in which the framework and the tool are utilised. Needless to say, in a real-life situation, the decision-maker(s) will iterate between the elicitation and evaluation steps, and also iterate repeatedly within the steps.

2. Representation model

Probabilistic decision situations are often described by a decision tree, as shown in Fig. 1.

The components of such a decision tree are a root node (also called a decision node), a set of probability nodes (representing uncertainty) and consequence nodes (the final outcomes). The probability and consequence nodes are in a standard model assigned unique numerical probability and value distributions. The semantics employed here are as follows: when an alternative A_i is selected, there is a probability p_{ij} that an event occurs that leads either to another subsequent event or to a consequence with a value v_{ijk}. A common evaluation rule in this context is the maximisation of the expected value; for instance, in the case of alternative A_i in Fig. 1, the expected value is:

$$E(A_i) = \sum_{j=1}^{2} p_{ij} \sum_{k=1}^{2} p_{ijk} v_{ijk1},$$

which can be straightforwardly generalised to the multi-linear equation:

$$E(A_i) = \sum_{i_1=1}^{n_{i_0}} p_{ii_1} \sum_{i_2=1}^{n_{i_1}} p_{ii_1i_2} \cdots \sum_{i_{m-1}=1}^{n_{i_{m-2}}} p_{ii_1i_2\cdots i_{m-2}i_{m-1}}$$

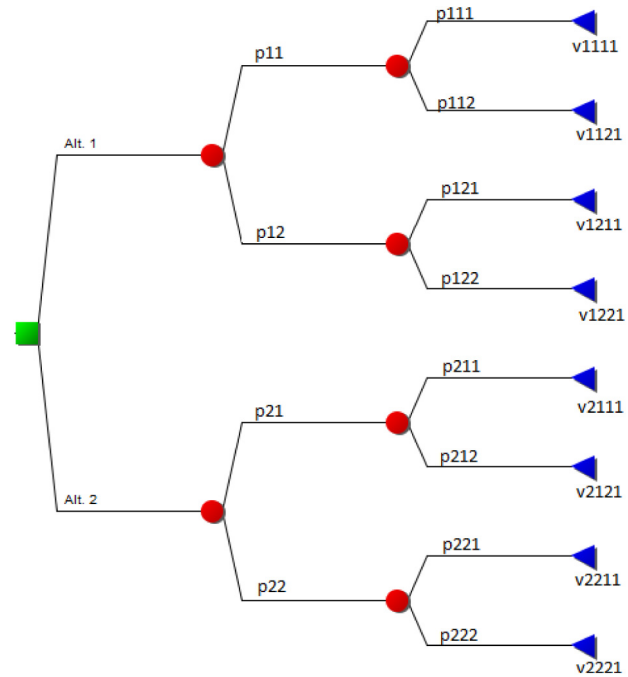

Fig. 1. A decision tree representation of a probabilistic decision situation.

$$\times \sum_{i_m=1}^{n_{i_{m-1}}} p_{ii_1i_2\cdots i_{m-2}i_{m-1}i_m} v_{ii_1i_2\cdots i_{m-2}i_{m-1}i_m 1},$$

where the p_{ij} with varying indices denote probabilities and the v_{ij} counterparts denote values.

This modelling can be generalised to cases where there is imprecise or incomplete information with respect to probabilities and consequence or alternative values, which we collect using the concept of a *multi-frame* to gather all necessary information. We discuss the theory for this below in an informal way.

User statements may be range constraints or comparative statements, which are translated into systems of inequalities in a *value constraint set*. Probability statements are collected in a *node constraint set*. User statements have the following forms:

- *Range constraints*: a probability or value y_i lies between a_1 and a_2, denoted as $y_i \in [a_1, a_2]$ and represented by $y_i > a_1$ and $y_i < a_2$ for real numbers a_1 and a_2.
- *Comparisons*: y_i is larger than y_j by a difference from d_1 to d_2, denoted $y_i - y_j \in [d_1, d_2]$ and represented by $y_i - y_j > d_1$ and $y_i - y_j < d_2$, for real numbers d_1 and d_2.

Constraint sets thus consist of linear inequalities and probability and value node constraint sets, characterising sets of (discrete) probabilities and distributions. Probability node constraint sets also involve the usual normalisation constraints ($\sum_j x_{ij} = 1$) that require the probabilities to sum to one.

Let T be a consequence tree and N be a constraint set for the variables { $n_{...i...j...}$ }. Then we substitute the intermediary node labels $x_{...i...j...}$ with $n_{...i...j...}$. N is then a *node constraint set* for T if for all sets $\{n_{...i1},\ldots, n_{...im}\}$ of all sub-nodes of nodes $n_{...i}$ that are not leaves, the statements $n_{...ij} \in [0,1]$ and $\sum_j n_{...ij} = 1$, $j \in [1,\ldots,m]$ are in N. We will also use the term *multi-frame* as a structure $\langle T, N\rangle$, where T is a consequence tree and N is a set of all constraints sets relative to T.

Given a set of variables $\{x_i\}_{i\in I}$ a *solution* to a system X of inequalities in $\{x_i\}_{i\in I}$ is a real vector $\mathbf{a} = (a_1,\ldots,a_n)$ where each a_i is substituted for x_i such that every inequality in the system

is satisfied. (There exists a solution if the substitution of a_i for x_i in X, for all $1 \leq i \leq n$, does not yield a contradiction). The vector **a** is called a *solution vector* to X. The *solution set* for X is {**b** | **b** is a solution to X}. A constraint set X in $\{x_i\}_{i \in I}$ is *consistent* **iff** the system of weak inequalities in X has a solution. A minimal requirement is that there must exist some vector of variable assignments that simultaneously satisfies each inequality in the constraint sets, i.e., given a consistent constraint set X in the variables $\{x_i\}$, ${}^{X}\max(x_i) =_{\text{def}} \sup(a \mid \{x_i > a\} \cup X$ is consistent). Similarly, ${}^{X}\min(x_i) =_{\text{def}} \inf(a \mid \{x_i < a\} \cup X$ is consistent). Furthermore, given a function *f*, ${}^{X}\text{argmax}(f(x))$ is a solution vector that is a solution to ${}^{X}\max(f(x))$, and ${}^{X}\text{argmin}(f(x))$ is a solution vector that is a solution to ${}^{X}\min(f(x))$. Finally, the set of orthogonal projections of the solution set is the *orthogonal hull*, more formally defined as the set of pairs $\langle {}^{X}\min(x_i), {}^{X}\max(x_i)\rangle$ given a consistent constraint set X in $\{x_i\}_{i \in [1,\ldots n]}$.

An orthogonal hull is thus straightforwardly found by solving a set of linear programming problems in standard fashion.

3. Introducing second-order beliefs

When specifying an interval, the actual beliefs in the values are probably not uniformly distributed. Earlier versions of the *Decide*IT tool handled such second-order belief approaches only to a limited extent in essentially two different ways, by contraction analysis and by Monte Carlo simulations. *Contraction analysis* was done by decreasing the interval widths by "contracting" the interval endpoints towards a focal point, the latter either being provided by the decision-maker or suggested as the centre of mass of the polytope defined by the intervals. Hence, the second-order representation was not explicit as the contraction analysis relied on the assumption that points closer to the centre of mass have larger belief mass compared to points closer to extreme points of the intervals. The amount of contraction (percentage of intervals cut off from the ends) required until the expression $\min[E(A_i) - E(A_j)] < 0$ was not satisfied in any single point of the remaining polytope (i.e. there existed no solution) was therefore viewed as a measure of robustness for a preference in favour of an alternative A_i compared to another alternative A_j. The level of contraction was given as a percentage of the original intervals but was restricted with respect to the granularity of the analyses. See, [30] for an extended presentation.

The Monte Carlo simulation approach took advantage of transformations between randomly generated points in a unit cube and a subset of the cube constructed from a user stipulated constraint set. A sampling algorithm was used for generating Dirichlet distributed probabilities [31] while a factorisation of a joint uniform distribution was used for generating ordered utility variables in the case of comparative constraints. See [32] for an account of this approach. The simulation approach could not allow both for upper probability bounds and for partial rankings of consequences and probabilities.

As a significant improvement over these earlier attempts, the tool now utilises belief distributions that indicate the strengths with which a decision-maker believes in these different values, and this approach is able to evaluate the results without any of the former constraints. The key differences can be seen in Table 1.

In this extended model, we first introduce parameters for belief distributions for probabilities and values; thereafter, we can operate on these distributions by utilising additive and multiplicative combination rules for random variables.[1]

Table 1
Key DecideIT functionality.

Functionality version	1.0	2.0	3.0
Probabilistic models	•	•	•
Probability intervals	•	•	•
Utility/value intervals	•	•	•
Multi-criteria models		•	•
Combined prob. and multi-criteria		•	•
Evaluation using contractions (first-order information)	•	•	(•)
Evaluation using belief mass (second-order information)			•

A unit cube is all tuples $(x_1, \ldots, x_n)$ in $[0,1]^n$ and a second-order distribution over such a cube *B* is a positive distribution *F* defined on *B* such that

$$\int_B F(x)dV_B(x) = 1, \text{ where } V_B \text{ is the n-dimensional Lebesgue measure on } B.$$

We use different distributions for probabilities and values because of the normalisation constraints for probabilities; natural candidates are the Dirichlet distribution for probabilities and two- or three-point distributions for values. The properties of the Dirichlet distribution as a parameterised family of continuous multivariate probability distributions make it suitable for this purpose.

The probability density function of the Dirichlet distribution is defined as

$$f_{dir}\,(p, \alpha) = \frac{\Gamma(\sum_{i=1}^{k} \alpha_i)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} {p_1}^{\alpha_i - 1} {p_2}^{\alpha_2 - 1} \ldots {p_k}^{\alpha_k - 1}$$

on a set $\{p = (p_1,\ldots p_k) \mid p_1,\ldots, p_k \geq 0,\ \Sigma p_i = 1\}$, where $(\alpha_1,\ldots, \alpha_k)$ is a parameter vector in which each $\alpha_i > 0$ and $\Gamma(\alpha_i)$ is the Gamma function.[2]

The Dirichlet distribution is thus a multivariate generalisation of the beta distribution, and the marginal distributions of Dirichlet are beta distributions. For instance, when a distribution is uniform, the marginal distribution is a polynomial of degree $n-2$, where n is the dimension of a cube B, i.e. when all $\alpha_i = 1$, then the Dirichlet distribution is uniform with the marginal distribution

$$f\,(x_i) = \int_{B_i^-} dV_{B_i^-}\,(x) = (n-1)(1-x_i)^{n-2}$$

For our purposes, we use a different form, namely the *bounded* Dirichlet distribution over a (normally user-specified) range instead of the interval [0,1]. Bounded beta distributions are then derived from this, giving four-parameter beta distributions. Thus, we define a *probability belief distribution* through a bounded Dirichlet distribution $f_3(a_i, c_i, b_i)$ where c_i is the estimated most likely probability and where a_i and b_i are the boundaries for the support of the distribution $(a_i < c_i < b_i)$ (cf. [38]).

For the values (i.e. without the normalisation constraint), the generalisation to a trapezoid is straightforward. A *delta distribution* is a two-point distribution (uniform or trapezoidal) or a three-point distribution (triangular). When we have no reason to make any other specific assumptions, for instance when there is large uncertainty in the underlying belief distributions involved, a two-point distribution modelling the upper and lower bounds (the uniform or trapezoid distributions) seems to be reasonable, even if this is seldom the case. However, when modal outcomes can be estimated to some extent, the beliefs would probably be better represented by three-point distributions. In this case, Beta and Erlang belief distributions generally give results similar

[1] The detailed background theory of belief distributions and aggregations in this sense is described in [30,33–36].

[2] The details of this are provided in any standard textbook on Bayesian statistics, such as [37].

to triangular distributions. Here, we assume that we only have limited sample data, but that the variable relationships are known in addition to the minima, maxima, and modal values. For instance, Golenko-Ginzburg [39] discusses PERT networks and their distributions. The mean value of a number of three-point value belief distributions $f_3(a_i, c_i, b_i)$ is $\mu(\lambda) = (a_i + b_i + \lambda c_i)/(\lambda + 2)$, with $\lambda = 1$ for triangular distributions and $\lambda = 0$ for a two-point uniform or trapezoid distribution.[3] For practical purposes, there is normally no reason to use any three-point distribution other than a triangular distribution, since the risk of underestimation is lower. When the decision data has been successfully elicited, the evaluation model can be applied to the data. This is covered in a later section, while the next section provides a brief review of our previous approaches to deal with second-order beliefs in the *Decide*IT software.

4. Evaluation model

The evaluation model is based on the resulting distribution over the generalised expected utility mentioned above, i.e.,

$$E(A_i) = \sum_{i_1=1}^{n_{i_0}} p_{ii_1} \sum_{i_2=1}^{n_{i_1}} p_{ii_1i_2} \cdots \sum_{i_{m-1}=1}^{n_{i_{m-2}}} p_{ii_1i_2\cdots i_{m-2}i_{m-1}} \times \sum_{i_m=1}^{n_{i_{m-1}}} p_{ii_1i_2\cdots i_{m-2}i_{m-1}i_m}\, v_{ii_1i_2\cdots i_{m-2}i_{m-1}i_m 1},$$

where we have distributions over the random variables p and v. Let G be a distribution over two cubes A and B, and assume that G has positive support for the feasible distributions at level i in a decision tree, and for the feasible probability distributions of the children of a node x_{ij}. Furthermore, assume that $f(x)$ and $g(y)$ are marginal distributions of G(z) on A and B, respectively. Then the cumulative multiplied distribution of the two belief distributions is

$$\mathrm{H}(z) = \iint_{\Gamma_z}^{f} (x)\, g(y)\, dxdy = \int_0^1 \int_0^{z/x} f(x)\, g(y)\, dxdy = \int_z^1 f(x)\, G(z/x)\, dx$$

where G is a primitive function of g, $\Gamma_z = \{(x, y) \mid x{\cdot}y \le z\}$, and $0 \le z \le 1$.

Let *h(z)* be the corresponding density function. Then

$$\mathrm{h}(z) = \frac{d}{dz}\int_z^1 f(x)\, G(z/x)\, dx = \int_z^1 \frac{f(x)\, g(z/x)}{x} dx.$$

Addition is handled by the standard convolution of two densities, restricted to A and B and the distribution h on a sum $z = x + y$, where we have the belief distributions $f(x)$ and $g(y)$ is consequently given by

$$\mathrm{h}(z) = \frac{d}{dz}\int_0^z f(x)\, g(z - x)\, dx.$$

Iterating this, we obtain the distribution over the generalised expected utility.

4.1. The B-normal method

The B-normal (business normal) method employs distributions that are qualified by assumptions made about the environment in which the method is going to be applied. These assumptions involve the frequency of the decisions made and the survival of the business over time. They allow methods from both risk and decision theory and business administration to be employed in forming the B-normal decision method.

In the same way as in standard risk and decision theory, we assume that a large number of events will occur and a large number of decisions will be made. In this way, maximising the expected value becomes a reasonable decision rule, and at the same time, the belief distributions over the expected values tend to normal distributions or distributions of a similar kind. Note that even when we assume that the expectations are estimated a large number of times (due to repeated decision making) and can consequently be approximated by a normal distribution, there are three observations in particular that should be considered here:

1. The resulting distributions will be approximately normal only when the original distributions are symmetric, which of course is not usually the case for beta and triangular distributions. The result will instead be skew-normal.
2. Even if the original distributions are symmetric, the non-linear multiplication operator breaks the symmetry. The result will again be approximately skew-normal.
3. To obtain a resulting approximate normal distribution, both the original distributions and their aggregations must allow for long tails. In general, this is not the case in our situation; this is because our estimates have lower and upper limits due to the fact that we use bounded Dirichlet distributions and uniform and triangular distributions, yielding approximately truncated normal distributions.

We therefore employ the skew-normal distribution to generalise the normal distribution by allowing for non-zero skewness, i.e. asymmetry. This is accomplished by introducing a shape parameter α, where $\alpha = 0$ represents the standard normal distribution, and $\alpha = 1$ yields the distribution of the maximum of two independent standard normal variates. We can then conveniently represent truncated (skew-)normal distributions as probability distributions of (skew-)normally distributed random variables that are bounded. The skewness of the distribution increases with the absolute value of α, and when $|\alpha| \to \infty$, we get folded normal or half-normal distributions. Distributions are right-skewed when $\alpha > 0$ and left-skewed when $\alpha < 0$. When the sign of α is changed, the density is reflected about $x = 0$. The skew-normal probability density function with location ξ, scale ω, and shape parameter α is

$$f(x) = \frac{2}{\omega}\varphi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right)$$

which turns into a standard normal distribution for $\alpha = 0$.

Assume that a distribution X has a normal distribution within the interval (a, b). Then X, $a < X < b$, has a truncated normal distribution and its probability density function is given by a four-parameter expression that tends to normality as the intervals are widened (see for instance [40]).

The B-normal distribution method employs a skew-normal distribution as above, adapting it to business use. It expresses the resulting distribution of outcomes of events in the context of business operations. The joint distribution of all modelled consequence pairs is approximately skew-normally distributed, and this approximation improves as the number of consequence pairs increases.

The adaptations of skew-normality to business use (B-normality) consist of:

- location and scale parameters that match the expected value and variance with the normal distribution while maintaining the same skewness;

[3] Beta-PERT usually uses $\lambda = 4$ and Erlang-PERT $\lambda = 3$. However, higher values of λ tend to underestimate the uncertainties involved.

- interpolated truncation toward the theoretical minimum and maximum expected values (i.e. the approximation tails extend further);
- handling of large skew, where standard skew-normality does not hold, by a successive limiting of the shape parameter;
- interpretation of binary risk trees as a going concern (the business operates an indefinite time into the future).

To employ the B-normal method, the skewed distribution must be aligned to give the same variance and expected value as its unskewed counterpart and must display the correct shape (skew). Assuming that the desired expected value is *E(X)*, the desired variance is σ^2, and the desired skew is *s*. The alignment (matching) of the B-normal distribution is done in three steps:

1. Obtain the shape parameter α that describes the desired skew *s* of a skew-normal distribution;
2. Once the shape parameter α is determined, this changes the variance of the B-normal distribution compared to a normal distribution. Adjust the scale parameter ω until the variance of the B-normal distribution is σ^2 and thus coincides with the corresponding normal distribution;
3. Since the shape and variance have been determined, this in turn changes the expected value of the distribution. To obtain the desired expected value *E(X)*, use the standard formula for the mean of a skew-normal distribution and solve for the location parameter ξ.

This procedure will yield the parameters α, ω^2, and ξ, and once these have been obtained, the B-normal distribution is parametrically determined. From this distribution, the belief (or confidence) in the different expected values can be determined in the same way as with standard normally distributed information. This is the core of the evaluation step using the B-normal distribution method. In real-life usage of the framework and the tool, a user will iterate repeatedly between the steps and phases within the steps. This paper merely describes the essentials of each step. Next, we try to illuminate the decision process supported by the tool using a real-life decision example from industry that was solved and decided using the *Decide*IT software tool. The example in the next section illuminates the discriminative power of second-order information since although the resulting alternative values are overlapping, we can quite strongly conclude a clear preference order.

5. Large-scale example

What sets *DecideIT* apart is the capacity to enter vague information in a large variety of ways and nevertheless get a solid result as is shown in the following example. The example is slightly simplified to not obscure the central features of the analyses. It is also anonymised but still reflects the essence of the original real-life case. It involves the management of a hydropower plant that experiences sensitive weeks in the late autumn when temperatures start to fall below 0 °C and the water freezes. At this stage, it is very important to let ice form with a thick enough surface, acting as a coating, to allow water to run free below it. Otherwise, there is a risk of frazil ice jamming the flow, possibly causing turbine failure, and also a risk of having to bypass the water flow, possibly causing flooding downstream and damage to critical infrastructure. However, allowing the ice to form a surface typically requires limiting power production, and this may result in large opportunity costs if the market price of electric power is high, which it typically is when the temperature drops. The decision problem thus involves whether to proceed with the production according to the normal sales plan, based upon certain prices (Alt. 1), or to reduce the runoff according to a pre-defined setting to allow an ice surface to form (Alt. 2).

Due to the multi-faceted consequences of frazil ice and floods, the firm considered the following seven evaluation criteria:

Cr. 1: Direct opportunity costs
Cr. 2: Indirect losses
Cr. 3: Power station safety
Cr. 4: Civil utility safety
Cr. 5: Public safety
Cr. 6: Local badwill
Cr. 7: Global badwill

For each criterion, three potential scenarios were defined, and these were modelled as three uncertain consequences, each corresponding to one scenario, with interval probabilities. Scenario 1 corresponded to the most likely scenario with no severe consequences, Scenario 3 corresponded to the worst-case scenario, and Scenario 2 was an in-between scenario akin to a previous situation that had occurred approximately ten years prior to the analysis. The probabilities were assessed by an expert panel consisting of on-site hydrologists and operations managers and were intentionally delivered with imprecision, due to a scarcity of historical data and the complexity of the causal effects of an increase in water level. Fig. 2 shows a screenshot of the software with the criteria model window, and Fig. 3 shows a decision tree for a criterion, with the outcomes for each scenario under the two alternatives. Thus, there was one such tree for each of the seven criteria.

The values of the consequences were either pointwise or interval estimates of the monetary costs the firm would incur should a particular scenario occur. The criteria weights were calibrated to yield a one-to-one trade-off such that, for example, one Euro in direct cost would equal one Euro in cost incurred from securing public safety. To sum up, the overall decision problem is shown in Table 2.

The weighted expected value intervals of the two alternatives then become

$$[\min(E(A_1)); \max(E(A_1))] = [-11; -1.4]$$
$$[\min(E(A_2)); \max(E(A_2))] = [-4.8; -0.2]$$

which clearly overlap. However, when additionally utilising second-order information (see below), we can make use of new analytical means for decision evaluation, foremost *support* and *remaining mass*, and here the new software features demonstrate their practical usability.

The *support* for alternative A_i as compared to alternative A_j is the joint belief mass where $E(A_i) > E(A_j)$. The *remaining mass* relies on the concept of *contraction*. Contraction analysis consists of shrinking the outer feasible boundaries of the expected value for each alternative while measuring $\max[E(A_i)-E(A_j)]$. The contraction level is indicated as a percentage, where at a 100% level of contraction, all feasible boundaries have been reduced to points (see [20]). The contraction level when $\min[E(A_i)-E(A_j)] > 0$ (or the complementary $\max[E(A_i)-E(A_j)] < 0$) is called the *intersection level*, and the joint belief mass remaining over the contracted orthogonal hull, when the intersection level is reached, is the remaining mass. The more mass that remains after the intersection level, the more confidence we can have in the final outcome of the analysis. See Fig. 4 for a so-called robustness graph evaluation showing these results for the current example. In the figure, the upper graph depicts the maximum difference $\max[E(A_1) - E(A_2)]$ and the bottom depicts $\min[E(A_1) - E(A_2)]$. These boundary graph lines form a cone that shrinks as the contraction increases, i.e. as the intervals are having more and more of their outer boundaries cut off toward a centre (focal) point. In the figure, it can be seen that as a result of the analysis 98% of the belief mass lies in the region where $E(A_1) - E(A_2)$

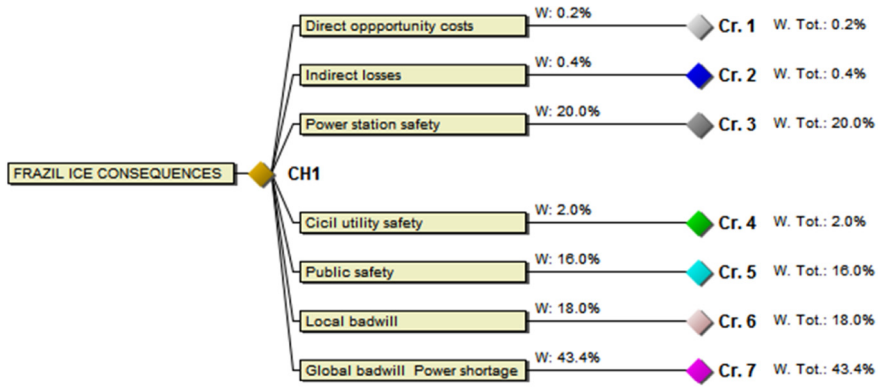

Fig. 2. Screenshot of software with window holding the criteria model. Beneath each node, the corresponding scenario decision tree is written out and can be accessed by a mouse click.

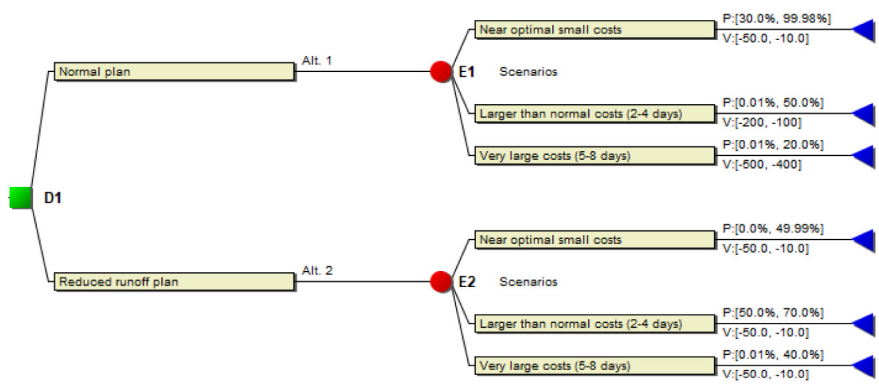

Fig. 3. Scenario decision tree for Criterion 1 showing direct opportunity costs, with interval probabilities and interval values for consequences.

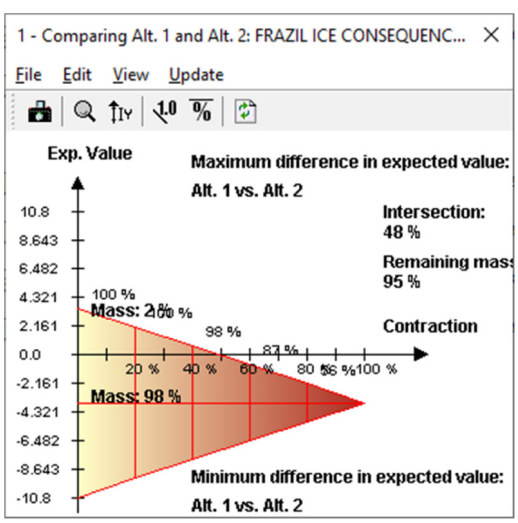

Fig. 4. Second-order decision evaluation. Screenshot from *Decide*IT software version 3.0.

< 0 which is a very confident result indeed and thus serves as a solid recommendation for a decision-maker. Furthermore, another result of the analysis is that 95% of the belief mass lies in the region to the right of the intersection with the x-axis, i.e. where $\max[E(A_1) - E(A_2)] < 0$ (remaining mass). Both of these results together point to a clear-cut result, which would not have been possible to obtain without exploiting second-order information in the decision evaluation.

Another evaluation in the *Decide*IT tool involves stacked bar charts of part-worth values for each criterion in combination with support information. The part-worth value φ_{il} for each alternative A_i under Criterion l is simply given by $\varphi_{il} = {}^c w_l \cdot {}^c v_{il}$ where ${}^c w_l$ and ${}^c v_{il}$ are the focal point weight for Criterion l and the focal point expected alternative value for alternative A_i under Criterion l, respectively. The height of each bar is then the sum $\varphi_{i1} + \varphi_{i2} + \cdots + \varphi_{in}$ for n criteria, which represents the aggregated value of each alternative. However, since this information is imprecise, the stacked bar charts are supplemented with support information, indicating confidence in the resulting ranking. 90% support is required for the result to be considered a confident result.

In Fig. 5, the height of the bars is normalised relative to the highest bar, which in this case is the bar for Alt. 2 which (as was seen in Fig. 3) has a significantly higher value in absolute terms. But in the figure, the value differences are relative to the minimum value of the selected main scale corresponding to "Direct cost" which is -500. Therefore, the bars seem to have similar heights, despite a substantial difference. The overall conclusion of the example is that Alt. 2, reduce the runoff to allow an ice surface to form, was clearly to prefer. This was also the action that was undertaken as a consequence of the decision analysis performed.

6. Concluding remarks

In real-life problems, it is usually impossible to assign precise numerical values to the different components of a decision model, and there is hence a need for representation and evaluation mechanisms that can effectively handle information incompleteness. Higher-order analyses such as belief mass can add both

Table 2
Criteria weights, scenario probabilities, and consequence values.

Criterion, weight	Alternative	Scenario 1	Scenario 2	Scenario 3
Cr. 1 $w_1 = 0.002$	Alt. 1	$p \in [0.3;1]$ $v \in [-50; -10]$	$p \in [0.01;0.5]$ $v \in [-200; -100]$	$p \in [0.01;0.2]$ $v \in [-500; -400]$
	Alt. 2	$p \in [0;0.5]$ $v \in [-50; -10]$	$p \in [0.5;0.7]$ $v \in [-50; -10]$	$p \in [0;0.4]$ $v \in [-50; -10]$
Cr. 2 $w_2 = 0.004$	Alt. 1	$p \in [0.12;0.23]$ $v \in [-50; -10]$	$p \in [0.75;0.85]$ $v \in [-200; -100]$	$p \in [0.02;0.03]$ $v \in [-400; -100]$
	Alt. 2	$p \in [0.39;0.6]$ $v \in [-50; -10]$	$p \in [0.4;0.6]$ $v \in [-200; -100]$	$p \in [0;0.01]$ $v \in [-1000; -500]$
Cr. 3 $w_3 = 0.2$	Alt. 1	$p \in [0.89;1]$ $v = 0$	$p \in [0.01;0.1]$ $v = -50$	$p \in [0;0.01]$ $v = -50\,000$
	Alt. 2	$p \in [0.97;0.99]$ $v = 0$	$p \in [0.01;0.02]$ $v = -50$	$p \in [0;0.01]$ $v = -50\,000$
Cr. 4 $w_4 = 0.02$	Alt. 1	$p \in [0.7;0.9]$ $v = 0$	$p \in [0.1;0.2]$ $v = -500$	$p \in [0.01;0.1]$ $v = -5000$
	Alt. 2	$p \in [0.93;0.98]$ $v = 0$	$p \in [0.01;0.05]$ $v = -500$	$p \in [0.01;0.02]$ $v = -5000$
Cr. 5 $w_5 = 0.16$	Alt. 1	$p \in [0.97;0.99]$ $v = 0$	$p \in [0.01;0.02]$ $v = -300$	$p \in [0;0.01]$ $v = -40\,000$
	Alt. 2	$p \in [0.99;1]$ $v = 0$	$p \in [0.97;0.99]$ $v = -300$	$p = 0$ $v = -40\,000$
Cr. 6 $w_6 = 0.18$	Alt. 1	$p \in [0.85;0.99]$ $v = 0$	$p \in [0.01;0.1]$ $v = -1000$	$p \in [0;0.05]$ $v = -45\,000$
	Alt. 2	$p \in [0.98;1]$ $v = 0$	$p \in [0;0.01]$ $v = -1000$	$p \in [0;0.01]$ $v = -45\,000$
Cr. 7 $w_7 = 0.43$	Alt. 1	$p \in [0.89;1]$ $v = 0$	$p \in [0;0.1]$ $v = -1500$	$p \in [0; 0.01]$ $v = -100\,000$
	Alt. 2	$p \in [0.97;1]$ $v = 0$	$p \in [0;0.02]$ $v = -1500$	$p \in [0; 0.01]$ $v = -100\,000$

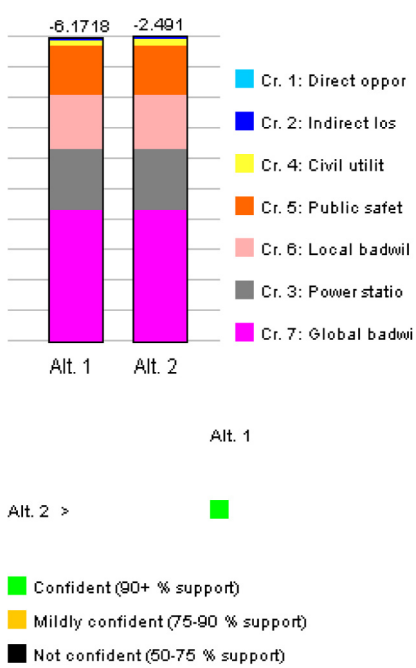

Fig. 5. Stacked bar chart evaluation showing the value contribution to an alternative from each criterion, together with the results of a support analysis showing that the support is 98%.

information and transparency, thus enabling a much more discriminative analysis than using intervals alone. We describe a higher-order framework realised by a software tool, based on an evaluation method using a belief mass interpretation of the data involved. We discuss a model and its implementation where second-order information is used for analysing both decision trees and a multi-criteria models, and as a demonstration, we apply it to an *actual real-life decision problem* from industry (power generation) to illustrate our new software features by demonstrating how second-order effects affect the resulting distribution over the expected values. The framework presented in this paper is put to use for solving decision situations using the tool that packages the framework and makes it available to real-life decision-makers with limited mathematical and decision theoretical knowledge, but an understanding of the decision context. Future work includes the design and development of efficient elicitation methods taking advantage of belief mass interpretations, and applications of the framework and accompanied software tools for analysing complex decision problems in business and public policy.

Acknowledgments

This research was supported by the EU-project Co-Inform (Co-Creating Misinformation-Resilient Societies H2020-SC6-CO-CREATION-2017) and strategic grants from the Swedish government within ICT - The Next Generation. This paper partially builds on an earlier short paper presented at the conference ISIPTA'19 in Gent, Belgium.

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Please cite this article as: M. Danielson, L. Ekenberg and A. Larsson, A second-order-based decision tool for evaluating decisions under conditions of severe uncertainty, Knowledge-Based Systems (2019) 105219, https://doi.org/10.1016/j.knosys.2019.105219.